Estimation of the Length and Orientation of the Line Between Two Closely Co-Orbiting Satellites

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The problem of estimating the baseline length and orientation using a combination of onboard sensor data and processed ground tracking data is considered. A minimum number (3) of simple onboard sensors is assumed. Some of the variables are assumed to be estimated from ground-tracking information only. These estimates become part of the observations, along with the onboard sensor outputs, for estimation of the remaining variables. A linear estimate of the remaining variables is derived via a Kalman filter. This sequential processing obviates the need for the simultaneous processing of ground track and onboard data. The importance of deciding which variables are to be estimated from the ground and which are to be estimated using the combined data is discussed. An example demonstrates that baseline length and orientation can be estimated within a few feet and fractions of a degree.

Nomenclature

B_0 = nominal base line length
b = baseline length
e = estimate error
H = observation matrix
K = filter gain
P,Q,R = covariance matrices of the estimate error, dynamic, and measurement noise, respectively
R_0 = nominal orbit radius
p,q = state vectors for satellites A, B, respectively
t = time
u,w = system dynamic noise
v = sensor noise
X,Y,Z = inertial axes
x^1,x^2 = state vectors
x_i = state variables
y = output vector
z = distance from nominal orbit plane
β = angle between baseline and nominal orbit plane
γ = angle of baseline in nominal orbit plane relative to inertial axes
η = angular location of B satellite with respect to A satellite
θ = orbital angular location of the satellite
μ = gravitational constant
ρ = correlation coefficient
σ = standard deviation
τ = sample interval
Φ = state transition matrix
Ψ = state vector for yaw dynamics
ψ = satellite yaw angle
ω_0 = orbital rate

Subscript

A, B = satellites A and B, respectively

Introduction

RECENTLY, interest has been expressed in using pairs and even clusters of satellites in close proximity to perform a variety of space missions.[1,2] One particular area in which dual satellite systems should prove very useful is in low-frequency radio astronomy. A pair of satellites may be used as an interferometer to synthesize a large aperture radio telescope.[2,3] As the pair moves in orbit, the location and frequency characteristics of various radio sources may be determined.

The satellites in these systems may be tethered,[4] thereby forming a very large "dumbbell" satellite, or they may be co-orbiting† but physically separate. In either case, however, when the pair of satellites is used as an interferometer, the length and orientation of the line between them (the baseline) must be determined to a relatively high degree of accuracy.

The problem considered here is the estimation of the baseline length and orientation of two separate but co-orbiting satellites. The basic assumption is that the baseline is small compared to the orbit dimension. For example, the radio astronomy mission may call for an altitude of 8000 miles and baseline of only one or two miles. Because of the closeness of the two satellites the baseline length and orientation cannot be adequately estimated from ground-based measurements alone. Small errors in estimates of the position vectors of each satellite reflect as large errors in the baseline orientation estimate. Thus, there is a requirement for some onboard sensors. The baseline length and orientation could be determined with an intersatellite ranging sensor and an optical sensor on one satellite viewing the second one against the star field background (the latter sensor determining baseline orientation). In this case onboard sensor data alone would solve the problem. An optical sensor of this type, however, does not presently exist. The approach here was to use simpler, state-of-the-art sensors. These sensors, however, do not provide enough information to make the system observable. The solution of the problem, therefore, lies in the combination of the onboard sensor data and ground tracking data.

The problem is formulated in terms of the small perturbations from the nominal circular orbit common to both satellites. The orientation of the baseline may be described by two angles, one in the nominal orbit plane and one between the baseline and this plane. The linearized equations for the in-plane and out-of-plane motions are not coupled. The problem is therefore reduced to two simpler ones. The state variables are defined as the sums and differences of the satellite positions and velocities. Some of the variables are estimated using processed ground-based measurements. These estimates, along with the onboard sensor information, are

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†Co-orbiting is taken to mean the two satellites are in the same nominal orbit, and the difference in the times from perigee is a constant.

used to estimate the remaining variables. The estimates of the two angles and baseline length are derived from the state variable estimates.

Problem Formulation

Assuming a circular Earth the inertial frame to be used as a reference is the XYZ frame where X and Y define the nominal orbit plane and the origin is at the center of the Earth (Fig. 1).

The orientation of the baseline relative to this frame may be described by the angles β and γ. Beta is the angle between the baseline and the X-Y plane and γ is the angle between the baseline planar projection and the X axis (Fig. 1). These angles and the baseline length b may be determined from the variables shown in the figure. Thus the estimates of γ, β, and b are functions of the vectors locating the satellites A and B in the XYZ frame.

The Eqs. of motion of each satellite with respect to this frame are

$$\ddot{R} - R\dot{\theta}^2 + \mu R/(R^2 + z^2)^{3/2} = 0 \quad (1a)$$

$$R\ddot{\theta} + 2\dot{R}\dot{\theta} = 0 \quad (1b)$$

$$\ddot{z} + \mu z/(R^2 + z^2)^{3/2} = 0 \quad (1c)$$

In Eqs. (1) cylindrical coordinates are used where R and θ are in the X-Y plane and z is the distance out of the plane. The gravitational constant is μ. The state variables for the A satellite are defined as components of the vector q. $q_1 = R_A$, $q_2 = R_0\theta_A$, $q_3 = \dot{R}_A$, $q_4 = R_0\dot{\theta}_A$, $q_5 = z_A$, $q_6 = \dot{z}_A$. The state variables for satellite B are similarly defined as components of the vector p. Components of the vectors q and p are indicated by numerical subscripts. The nominal orbit radius is R_0. When the Eqs. are linearized with respect to the nominal circular orbit, the system Eqs. are

$$\delta q(t) = \Phi(t,t_0)\delta q(t_0) + u(t) \quad (2a)$$

$$\delta p(t) = \Phi(t,t_0)\delta p(t_0) + u(t) \quad (2b)$$

$$\delta y(t) = H(t)\begin{bmatrix}\delta q(t)\\ \delta p(t)\end{bmatrix} + v(t) \quad (3)$$

In Eqs. (2) and (3) $\delta q(t)$ and $\delta q(t)$ are the first variations of $q(t)$ and $p(t)$. The dynamic noise is $u(t)$ (the same for both satellites because of their proximity) and the sensor noise is $v(t)$. The state transition matrix $\Phi(t,t_0)$ is listed in the Appendix. The lack of coupling between the planar variables, δq_1, δp_1–δq_4, δp_4 and the nonplanar variables $\delta q_5,\delta p_5,\delta q_6,\delta p_6$ is evident.

To determine the relationship between the output vector $\delta y(t)$ and the state vectors the matrix $H(t)$ must be defined. This requires definition of the sensors. The following sensors are assumed to be located on A: 1) an intersatellite range sensor measuring baseline length b; 2) a yaw sensor measuring the A satellite yaw angle ψ (the attitude angle around the orbit radius vector); 3) An optical "B-tracker" locating the B satellite with respect to the A satellite attitude reference frame. This measures the angle η. The latter two angles are shown in Fig. 2. Both ψ and η are nominally zero. The η sensor is assumed to be independent of the small variation in γ.

The relation between the state variables and the observables $[\delta y(t)]$ may be determined from the geometry of the problem. As a function of the state variables, b is

$$b = \{q_1^2 + p_1^2 - 2q_1p_1\cos[(q_2 - p_2)/R_0] + (q_5 - p_5)^2\}^{1/2} \quad (4)$$

The first variation in b is therefore

$$\delta b = (\delta q_1 + \delta p_1)\sin(\Delta\theta/2) + (\delta q_2 - \delta p_2)\cos(\Delta\theta/2) \quad (5)$$

where $\Delta\theta$ is the angle subtended by the nominal baseline B_0 when A and B are in the nominal orbit, i.e., $\sin(\Delta\theta/2) = B_0/2R_0$. Equation (5) is independent of the z variables,

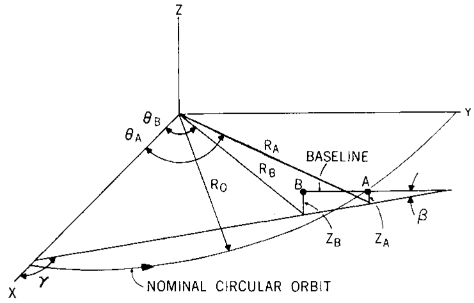

Fig. 1 Orbital configuration of the dual satellite system.

δq_5 and δp_5. The intersatellite range sensor is the only onboard sensor required for the estimation of b and γ. That the estimate of b may be derived from this information is obvious. It will be shown that the γ estimate may also be derived using only the intersatellite range information (b measurement) and processed ground tracking information.

The angle β is expressed as a function of the state variables as follows

$$\beta = \sin^{-1}((p_5 - q_5)/\{q_1^2 + p_1^2 - 2q_1p_1 \times \cos[(q_2 - p_2)/R_0] + (p_5 - q_5)^2\}^{1/2}) \quad (6)$$

This angle is not directly observable but is related to the angles ψ,η (Fig. 2)

$$\beta = \eta - \psi \quad (7)$$

Using Eqs. (6) and (7) the first variations in the observables ψ and η are $\delta\psi$ and

$$\delta\eta = \delta\psi + (1/B_0)(\delta p_5 - \delta q_5) \quad (8)$$

The introduction of $\delta\psi$ requires the introduction of the linearized yaw attitude dynamics

$$\delta\Psi(t) = \Phi_3(t,t_0)\delta\Psi(t_0) + w(t) \quad (9)$$

where $\delta\Psi(t)$ is the $r \times 1$ state vector with $\delta\psi$ as its first component and $w(t)$ is the dynamic noise. The corresponding transition matrix is $\Phi_3(t,t_0)$.

Examination of Eqs. (5) and (8) shows that planar and nonplanar observations are uncoupled just as the dynamics. The problem may therefore be considered as two uncoupled problems. Equations (2) and (3) hold for the planar problem [but only the first four components of δq and δp are used and $\Phi_1(t,t_0)$ replaces $\Phi(t,t_0)$]. In the planar case $\delta y(t) = \delta\tilde{b}(t)$, $H = [\sin(\Delta\theta/2), \cos(\Delta\theta/2),0,0, \sin(\Delta\theta/2), -\cos(\Delta\theta/2),0,0]$ and $v_1(t)$ is the range sensor noise. The tilde ($\sim$) refers to the observable with measurement noise included.

The corresponding equations for the nonplanar problem are Eqs. (10–12).

$$\begin{bmatrix}\delta\Psi(t)\\ \delta q_5(t)\\ \delta q_6(t)\\ \delta p_5(t)\\ \delta p_6(t)\end{bmatrix} = \begin{bmatrix}\Phi_3(t,t_0) & 0 & 0\\ 0 & \Phi_2(t,t_0) & 0\\ 0 & 0 & \Phi_2(t,t_0)\end{bmatrix}\begin{bmatrix}\delta\Psi(t_0)\\ \delta q_5(t_0)\\ \delta q_6(t_0)\\ \delta p_5(t_0)\\ \delta p_6(t_0)\end{bmatrix} + \begin{bmatrix}w(t)\\ u_5(t)\\ u_6(t)\\ u_5(t)\\ u_6(t)\end{bmatrix} \quad (10)$$

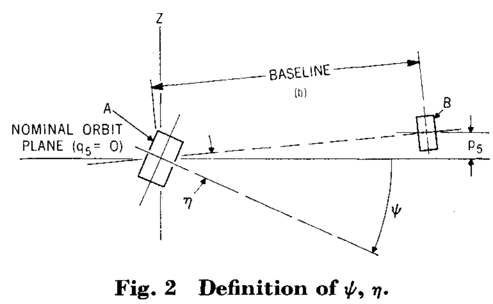

Fig. 2 Definition of ψ, η.

$$\delta y(t) = \begin{bmatrix} \delta\tilde{\psi}(t) \\ \delta\tilde{\eta}(t) \end{bmatrix} = H_1 \begin{bmatrix} \delta\Psi(t) \\ \hdashline \delta q_5(t) \\ \delta q_6(t) \\ \delta p_5(t) \\ \delta p_6(t) \end{bmatrix} + \begin{bmatrix} v_2(t) \\ v_3(t) \end{bmatrix} \quad (11)$$

$$H_1 = \Big[\underbrace{\begin{matrix} 1 & 0 & \cdots & 0 \\ 1 & 0 & \cdots & 0 \end{matrix}}_{r} \Big| \begin{matrix} 0 & 0 & 0 & 0 \\ -1/B_0 & 0 & -1/B_0 & 0 \end{matrix}\Big] \quad (12)$$

The ψ and η sensors have noise components $v_2(t)$ and $v_3(t)$, respectively. The components of $v(t)$ are all assumed to have zero mean.

Planar Solution

Estimates of the variables b and γ may be determined as functions of the estimates of the planar state variables. For b the relationship has already been shown to be Eq. (5). The first variation in γ is

$$\delta\gamma = -(1/B_0)(\delta q_1 - \delta p_1) + (1/2R_0)(\delta q_2 + \delta p_2) \quad (13)$$

As the problem stands there are eight variables $\delta q_i, \delta p_i, i = 1,4$ and one observable $\delta\tilde{b}$. With the single onboard sensor, the system is not observable. Not enough information is available to allow an estimate of δq and δp. This problem may be overcome by redefining the state variables as sums and differences of the δq_i's and δp_i's and then estimating some of the new variables from ground observations.

Let

$$x_i = \begin{cases} \delta q_i - \delta p_i & i = 1,6 \quad (14a) \\ \delta q_{i-6} + \delta p_{i-6} & i = 7,12 \quad (14b) \end{cases}$$

With this change of variables the planar system Eqs. become

$$x^1(t) = \Phi_1(t,t_0)x^1(t_0) \quad (15)$$

$$x^2(t) = \Phi_1(t,t_0)x^2(t_0) \quad (16)$$

$$\delta\tilde{b} = x_2 \cos(\Delta\theta/2) + x_7 \sin(\Delta\theta/2) + v_1(t) \quad (17)$$

where $x^1 = [x_1,x_2,x_3,x_4]^T$ and $x^2 = [x_7,x_8,x_9,x_{10}]^T$.‡

In order to provide the necessary information to make the system observable it is now assumed x^2 is estimated from ground observations. Thus Eq. (16) is no longer needed. Note also that $\delta\tilde{b}$ contains components from both x^1 and x^2.

Denote the estimate of x_i by $\hat{x}_1$ and the error in the estimate by e_i.

$$x_i = \hat{x}_i - e_i \quad i = 1,12 \quad (18)$$

Substituting for x_7 in Eq. (17) results in

$$\delta\tilde{b} = x_2 \cos(\Delta\theta/2) + \hat{x}_7 \sin(\Delta\theta/2) - e_7 \sin(\Delta\theta/2) + v_1(t) \quad (19)$$

At any time the estimate $\hat{x}_7$ is a constant entered from an external source (the output of the x^2 estimation process). Thus, it may be incorporated in the measurement $\delta\tilde{b}$. The statistical properties of e_7 and $v_1(t)$ are assumed to be known, and e_7 and $v_1(t)$ are uncorrelated. Taken together these terms constitute the total measurement noise. The output equation therefore reduces to

$$\delta\tilde{b} = [0, \cos(\Delta\theta/2),0,0]x^1 - e_7 \sin(\Delta\theta/2) + v_1(t) \quad (20)$$

Equations (15) and (20) are in the correct form to allow x^1 to be estimated.

The vector x_1 may be estimated by means of the Kalman filter equations[5];

$$\hat{x}(t+1) = \Phi_1(t+1,t)\hat{x}(t) + K(t+1)[\delta y(t+1) - H(t+1)\Phi_1(t+1,t)\hat{x}(t)] \quad (21)$$

$$K(t+1) = P_t(t+1)H^T(t+1) \times [H(t+1)P_t(t+1)H^T(t+1) + R(t+1)]^{-1} \quad (22)$$

The covariance matrix of the estimate error $P(t+1)$ is determined by Eqs. (23) and (24);

$$P_t(t+1) = \Phi_1(t+1,t)P(t)\Phi_1^T(t+1) + Q(t) \quad (23)$$

$$P(t+1) = P_t(t+1) - K(t+1)H^T(t+1)P_t(t+1) \quad (24)$$

In Eqs. (21–24), $\hat{x}(t+1)$ is the estimate given measurements through time $t+1$, $\hat{x}(t)$ is the estimate given measurements through time t. The measurement interval has been normalized to one in these equations. The covariance matrices of the dynamic noise and measurement noise are $Q(t)$ and $R(t)$, respectively. For the planar problem

$$\begin{aligned} R(t) &= E\{[v_1(t) - e_7 \sin(\Delta\theta/2)]^2\} \\ &= \sigma_{v1}^2 + \sigma_7^2 \sin^2(\Delta\theta/2) \end{aligned} \quad (25)$$

In Eq. (25) $E\{\ \}$ is the expected value operator and $\sigma_{v1}^2, \sigma_7^2$ are the variances of the range sensor noise and x_7 estimate error, respectively. Since $v_1(t)$ and e_7 are uncorrelated $E\{v_1(t)e_7\} = 0$.

By using Eqs. (13) and (14) and $\hat{x}^1$ and $\hat{x}^2$, $\delta\gamma$ may be estimated

$$\delta\hat{\gamma} = -(1/B_0)\hat{x}_1 + (1/2R_0)\hat{x}_8 \quad (26)$$

In Eq. (26) $\hat{x}_1$ is from $\hat{x}^1$, $\hat{x}_8$ is from $\hat{x}^2$ and $\delta\hat{\gamma}$ is the estimate of the variation $\delta\gamma$. The error of the estimate is $e_\gamma = \delta\hat{\gamma} - \delta\gamma$. The variance of the estimate error is

$$\sigma_\gamma^2 = \sigma_1^2/B_0^2 + \sigma_8^2/4R_0^2 - \rho\sigma_1\sigma_8/R_0B_0 \quad (27)$$

where σ_1^2, σ_8^2 are the variances of the $\hat{x}_1$ and $\hat{x}_8$ estimate errors and ρ is the correlation coefficient between these estimate errors. Because of the separate processing of $\hat{x}^1$ and $\hat{x}^2$, ρ is not directly available. However σ_γ^2 is bounded by

$$\sigma_{\gamma*}^2 = \sigma_1^2/B_0^2 + \sigma_8^2/4R_0^2 + \sigma_1\sigma_8/R_0B_0 \quad (28)$$

Because of the basic assumption that $B_0 \ll R_0$ it will turn out that $\sigma_{\gamma*} \cong \sigma_\gamma$. This will be demonstrated in the example.

The estimate of δb and variance of the estimate error are

$$\delta\hat{b} = H\hat{x}^1 = \cos(\Delta\theta/2)\hat{x}_2 \quad (29a)$$

$$\sigma_b^2 = \cos^2(\Delta\theta/2)\sigma_2^2 \quad (29b)$$

Nonplanar Solution

The angle β is defined as a function of the state variables in Eq. (6). With the variables defined in Eq. (14) the first variation is

$$\delta\beta = -x_5/B_0 \quad (30)$$

Thus the estimate of β is a function of the nonplanar variable x_5 only.

The system equations for this problem using Eqs. (10–12) are

$$\begin{bmatrix} \delta\Psi(t) \\ \hdashline x_5(t) \\ x_6(t) \end{bmatrix} = \begin{bmatrix} \Phi_3(t,t_0) & 0 \\ 0 & \Phi_2(t,t_0) \end{bmatrix} \begin{bmatrix} \delta\Psi(t_0) \\ \hdashline x_5(t_0) \\ x_6(t_0) \end{bmatrix} + \begin{bmatrix} w(t) \\ \hdashline 0 \\ 0 \end{bmatrix} \quad (31)$$

$$\begin{bmatrix} \delta\tilde{\psi} \\ \delta\tilde{\eta} \end{bmatrix} = \begin{bmatrix} 1 & 0 & \cdots & 0 & 0 & 0 \\ 1 & 0 & \cdots & 0 & 1/B_0 & 0 \end{bmatrix} \begin{bmatrix} \delta\Psi(t) \\ \hdashline x_5(t) \\ x_6(t) \end{bmatrix} + \begin{bmatrix} v_2(t) \\ v_3(t) \end{bmatrix} \quad (32)$$

‡ The T indicates transpose.

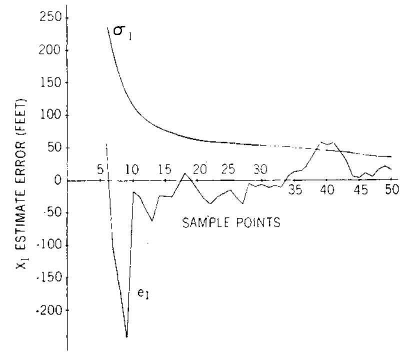

Fig. 3 Estimate error and standard deviation of $\hat{x}_1$.

Once an estimate of x_5 is available (via the filter equations) the estimate of $\delta\beta$ and variance of the estimate error are, respectively,

$$\delta\hat{\beta} = -\hat{x}_5/B_0, \quad \sigma\beta^2 = \sigma_5{}^2/B_0{}^2 \tag{33}$$

Example

Consider a dual satellite system with the mission of low-frequency radio astronomy. The co-orbiting satellites are the receivers of an orbiting interferometer. In order to determine the location and frequency characteristics of the emitting sources the baseline length (b) and orientation (γ,β) must be known.

A digital simulation was carried out using the following parameters: $R_0 = 7 \times 10^7$ ft, $\omega_0 = 2.03 \times 10^{-4}$ sec^{-1}, $B_0 = 10^4$ ft, and $\tau = 100$ sec (sampling interval). The dynamic noise $[u(t)]$ was assumed to be zero. The measurement noise $[v(t)]$ was assumed to be white Gaussian with zero mean and standard deviations: $\sigma_{v1} = 5$ ft, $\sigma_{v2} = \sigma_{v3} = 8.7$ mrad (0.5°). The estimate errors of $\hat{x}_7$ and $\hat{x}_8$ after processing of ground based measurements were assumed to have the following standard deviations: $\sigma_7 = 1500$ ft, $\sigma_8 = 300$ ft. These standard deviations (σ_7,σ_8) are not the result of a particular study but are reasonable numbers for radius and orbit track errors given previous spacecraft histories. They include all the effects of tracking station errors, location, etc.

With Eqs. (15) and (20) defining the system, Eqs. (21–24) were used to estimate x^1. $Q(t)$ is zero because the dynamic noise was assumed to be zero. $R(t)$ is defined in Eq. (25). Since $\sin(\Delta\theta/2) = B_0/2R_0$ and $B_0 \ll R_0$, the second term of $R(t)$ is small compared to the first, i.e., the ground-based measurement errors had little effect on this estimate. Sample runs are shown in Figs. 3 and 4. These figures show the estimate errors of $\hat{x}_1$ and $\hat{x}_2$ (i.e., e_1 and e_3) and their respective standard deviations. Good estimates of x_1 and x_2 are available after 50 samples (5000 sec). Once the estimates of x^1 and x^2 are available, the planar variables of interest γ and b may be estimated with Eqs. (26) and (29). Using σ_1 and σ_2 after 50 samples (Figs. 3 and 4) and the assumed σ_8, the variances of the γ and b estimate errors may be found with Eqs. (27) and (29b). With σ_1 and σ_2 of 35 ft and 2.5 ft, respectively, $\sigma_{\gamma^*} = 35 \times 10^{-4}$ rad (0.2°) and $\sigma_b = 2.5$ ft. Because $B_0 \ll R_0$ the last two terms in Eq. (28) are negligible, i.e., the ground-based measurement errors had little effect on the estimates of γ and b.

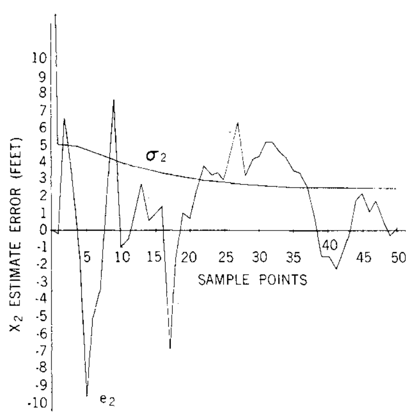

Fig. 4 Estimate error and standard deviation of $\hat{x}_2$.

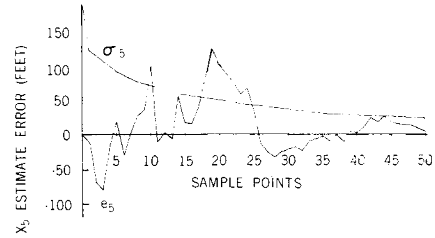

Fig. 5 Estimate error and standard deviation of $\hat{x}_5$.

For the nonplanar case where the angle β is of interest, Eqs. (31) and (32) define the system. A simple second order system was assumed for the yaw attitude dynamics $[\Phi_3(t,t_0)]$. Equations (21–24) are used to estimate the state vector which is defined on the left side of Eq. (31). Once this vector is estimated the β estimate and variance of the estimate error are found using Eq. (33).

Figure 5 shows a sample run. The resulting σ_β after 50 samples is 25×10^{-4} rad (0.14°).

It is important to note that although the ground-based measurement errors had little effect on these estimates, they would have had a large effect if ground measurements only were used. With errors on the order of σ_7,σ_8 the variables γ and b could not be estimated as accurately as demonstrated in this example.

Discussion

Two points should be emphasized with respect to the preceding results. There may be cases in which the β estimate is not required either because the out-of-plane motion is not significant or because the sources of interest are in or near the orbit plane. In this case the quantities of interest γ and b may be estimated with a single on board intersatellite range sensor; no optical angle sensors (B-trackers) are required on board the satellites.

Because the onboard sensors provide only part of the information required to make the system observable it was necessary to assume other information was available. This was done by redefining the variables [Eq. (14)] and assuming x^2 was estimated from ground based measurements. Instead of doing this it could have been assumed that one of the vectors q or p is estimated from the ground. This would provide enough information to allow an estimate of the other vector. However, the large uncertainty in the radius variation δq_1 or δp_1 reflects directly into $\delta\hat{\gamma}$ as evidenced by Eq. (13). The resulting σ_γ would be much larger than σ_γ as determined by the described method. Thus, the choice of the variables to be estimated is important. Since only the sums and differences of the variables are required to estimate $\delta\gamma$, δb, and $\delta\beta$, only these are estimated and estimation of the absolute variations $\delta q_1,\delta p_1$ is avoided.

Conclusion

The three parameters describing the length and orientation of the line between two closely co-orbiting satellites cannot be adequately estimated from ground measurements alone because small uncertainties in the satellite location vectors contribute to large uncertainties in the baseline orientation. A minimum number of simple onboard sensors alone cannot do the job completely because they do not provide enough information in themselves. However, by combining the information provided by three onboard sensors (two angle sensors and an intersatellite ranging system) with the processed ground tracking information, a good linear estimate of these three parameters can be obtained. Care must be taken in defining the variables to be estimated to insure a low error in the final output.

Appendix

The state transition matrix $\Phi(t,t_0)$ is

$$\Phi(t,t_0) = \left[\begin{array}{c|c} \Phi_1(t,t_0) & 0 \\ \hline 0 & \Phi_2(t,t_0) \end{array}\right] \quad \text{(A1)}$$

$$\Phi_1(t,t_0) = \begin{bmatrix} (4 - 3\cos\omega_0\tau) & 0 & (\sin\omega_0\tau)/\omega_0 & 2(1 - \cos\omega_0\tau)/\omega_0 \\ 6(\sin\omega_0\tau - \omega_0\tau) & 1 & 2(\cos\omega_0\tau - 1)/\omega_0 & (4\sin\omega_0\tau - 3\omega_0\tau)/\omega_0 \\ 3\omega_0\sin\omega_0\tau & 0 & \cos\omega_0\tau & 2\sin\omega_0\tau \\ 6\omega_0(\cos\omega_0\tau - 1) & 0 & -2\sin\omega_0\tau & (4\cos\omega_0\tau - 3) \end{bmatrix} \quad \text{(A2)}$$

$$\Phi_2(t,t_0)\begin{bmatrix} \cos\omega_0\tau & (\sin\omega_0\tau)/\omega_0 \\ -\omega_0\sin\omega_0\tau & \cos\omega_0\tau \end{bmatrix} \quad \text{(A3)}$$

$$\tau = t - t_0 \quad \text{(A4)}$$

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Use of Venus Gravitational Field for Solar Probe Trajectory Control

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The possibility of use of braking capabilities of the Venus gravitational field for probe trajectory control is discussed. The control problem consists in minimizing the probe flight time from the launch moment up to the moment of probe eclipse by the sun. The Earth and Venus coordinates are computed over the mean Newcomb elements. The probe flight trajectory is considered to be Keplerian and radius of the Venus sphere action is held to be a negligible. On the basis of the analysis, main characteristics of the flight trajectories as well as their interrelations are determined. The efficiency of using the Venus gravitational field is evaluated according to the velocity required for placing the probe into the flight trajectory and for trajectory correction.

Nomenclature

$a_\odot$ = semimajor axis of probe's heliocentric orbit after Venus flyby

$a,r,\mathbf{V}$ = semimajor axis of Venus orbit, distance from Venus to the sun, and Venus velocity vector at the moment of probe's impact of Venus, respectively

R = Venus radius

$t_\odot$ = probe transfer time from the Earth to the moment of probe eclipse by the sun

$t_♀$ = probe transfer time from the Earth to Venus

$t_☊,t_☋$ = probe transfer time from the Earth to a probe orbit ascending and descending ecliptic nodes, respectively

$t_{\oplus☊},t_{\oplus☋}$ = Earth transfer time from departure date to probe orbit ascending and descending ecliptic node, respectively

$T_\odot,T_\oplus$ = sidereal and synodic periods of a probe's heliocentric orbit after Venus flyby, respectively

$\text{grad}_1 T_\odot,\text{grad}_2 T_\odot$ = gradients of probe orbit period at the moments of both the first and the second corrections in probe velocity at the same moments, respectively

$\text{grad}_1 t_☊$ = gradient of probe flight time from the first correction moment up to the moment of the eclipse node passing in the probe velocity at the first correction moment

$V_♀$ = velocity of a probe with respect to Venus upon the sphere of its action

$\mathbf{V}_{♀1},\mathbf{V}_{♀2}$ = vectors of probe velocity with respect to Venus upon the sphere of its action before and after Venus flyby, respectively

$V_\oplus$ = velocity of a probe with respect to the Earth upon the sphere of its action

V_ξ,V_η = projections of Venus velocity vector along $O\xi$ and $O\eta$ axes, respectively

$\Delta\rho$ = maximum deviation of a point of impact from an aiming point on target plane

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